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The evolution of the mass of a "preserved" chemical spill as it moves with the river current☆

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ABSTRACT

A quantitative model describing the process of turbulent diffusion of the liquid mass of a chemical spill as it moves with the river current is constructed. The model contains a representation of the tensor components of the turbulent diffusion coefficients in terms of the initial parameters of the problem – the breadth and depth of the water flow in the river and the average flow velocity. According to the model proposed, the process consists of three stages: the initial, rapid stage, in which, owing to turbulent diffusion along the vertical coordinate, the pollutant concentration distribution evens out along this coordinate; the intermediate stage, in which, because of turbulent diffusion along the horizontal coordinates, the concentrations are likewise evened out along the horizontal coordinate transverse to the river channel; and the third, longest and slowest stage, in which quasi-one-dimensional turbulent diffusion occurs along the longitudinal (channel) coordinate, describing the "spreading" of the chemical slick (volume) carried downstream. Simple explicit formulae are obtained for a quantitative estimation of the characteristic pollutant concentrations at the end of the first and second stages and their "attenuation" with increasing distance as the chemical slick drifts with the current, and also the increasing longitudinal dimension of the slick with distance downstream.

The industrial development of civilisation on Earth has been beset with a host of technogenic accidents, occasionally catastrophic, including the discharge into rivers of toxic chemicals that are dangerous to living creatures. A typical example is the recent accident at a chemical factory in the Chinese town of Jilin, as a result of which about 100 m^3 of polluted liquid was spilt into the Songhua River– the largest tributary of the Amur River.

Carried along by the river current, the polluted liquid mass is gradually mixed, by turbulent transfer, with the river water and diluted by it, and the concentration of toxic chemicals decreases.

It is of considerable ecological interest to forecast the changes to the pollutant – the changes in its concentration in an increasing volume of polluted water – as it flows downstream. The results of a quantitative estimation of these changes determine the decisions taken concerning the different technical and organizational measures to protect the population of towns and cities and other settlements downstream of the spill that require river water for drinking and other needs.

A detailed forecast is possible only by constructing a mathematical model of the process of drift of the polluted volume of water with the current, its mixing with the river water and its "spread" in size, with a resultant fall, with time and along spatial coordinates, in the concentrations of polluting impurities. Such a model will be constructed in the present paper, and estimates will be made for the Songhua–Amur incident.

The water current in rivers is turbulent – against a background of low average flow velocities (in large lowland rivers), because of the gradients of these flow velocities in the vertical and horizontal directions, different-scale turbulent motions of the water arise, and this generates the relative transfer of individual water masses in the directions indicated and their mixing with the surrounding masses of water, and turbulent diffusion of impurities contained in the water occurs. A quantitative description of these processes is a very complex problem: both the turbulent current and the transfer and mixing of impurities that occur because of it are very non-linear processes for which there are no precise mathematical models. Approximate models exist, based on different hypotheses concerning the "structure" of the turbulence, the adequacy of which is "adjusted" in specific situations by comparisons with given observations. Some idea of the state of the art in these matters can be gleaned elsewhere.[1–4]

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To describe turbulent diffusion, the molecular diffusion coefficient in the normal diffusion equation is replaced by the tensor of the turbulent diffusion coefficients, the determination of which requires appropriate hypotheses.

We will use this approach. Then, for the case of a "preserved" pollutant, when it is not "removed" from the water mass by processes of adsorption on the surface of the river bed or by chemical and biochemical interactions with the water, leading to the formation of substances sinking or surfacing in the water, the equation of turbulent diffusion can be written in the form

$$\frac{\partial C}{\partial t} + \upsilon\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left(K_x\frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y\frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z\frac{\partial C}{\partial z}\right), \tag{1}$$

where C is the impurity concentration, $x = s - Vt$, y and z are the spatial coordinates, s along the river bed, y transverse to it in the horizontal direction and z along the vertical (in spite of the curvilinearity of this coordinate system, because the curvature of the channel axis is relatively small, the notation of the diffusion equation in form (1) is fairly precise), t is the time, V is the flow rate averaged over the cross-section of the water flow, υ is the average velocity (without turbulent pulsations) along the channel in the x, y, z system (the analogous rates along the y and z axes are zero), and K_x, K_y and K_z are the coefficients of turbulent diffusion, taken to be constant below. This last assumption is the most crude; it is used in order to be able to obtain visible estimates of the orders of magnitude of the parameters analysed in the problem. It is this that makes the diffusion equation (1) linear and enables us to make a complete analysis of the process.

For a "preserved" pollutant not adsorbed by the river bed, river banks or aquatic life, the three-dimensional equation (1) must be solved under boundary conditions expressing the absence of flows of this substance through boundaries (the bed, the banks and the free surface of the water).

Similar modelling of turbulent effects has been used in the literature to investigae a number of problems in oceanology (for example, in the book edited by A. S. Monin[5]) and gives acceptable results. However, no rational approaches have been used to determine a priori of the dependence of the turbulent diffusion coefficients on the governing parameters of the problem. This is a very considerable deficiency of existing studies of this problem. Here, we will represent this dependence by approximate relations and make a quantitative assessment. We first remark that, generally speaking, the tensor of the turbulent diffusion coefficients does not need to have a diagonal form, as adopted in the writing of Eq. (1). However, an attempt to determine the values of off-diagonal coefficients K_{xy}, K_{yz} and K_{xz}, to assess the effect of taking them into account when solving of the problem, is unwarranted because, on the one hand, there is as yet no reliable theory to support this, and, on the other hand, taking these coefficients into account in reasonable estimates of their orders of magnitude cannot appreciably alter the forecasts obtained using model (1), whereas doing so will greatly complicate the model itself.

To estimate a priori the dependence of the coefficients K_x, K_y and K_z on the governing parameters of the problem, we will proceed from the fact that turbulence only arises when there are gradients of the average velocities along the spatial coordinates. In the present problem there are three characteristic gradients of this kind – along the x, y and z coordinates. The orders of magnitude of these parameters are estimated from the ratios V/L_x, V/L_y and V/L_z, where L_x, L_y and L_z are the characteristic linear dimensions of the problem. For a river, $L_y \sim D$ and $L_z \sim h$, where D and h are the characteristic breadth and depth of the water flow. As regards L_x, it can be assumed that the estimate here will be of the order of $L_x \sim kD$, $k \sim 1$–6, because diffusion in the x direction will chiefly be determined by the velocity gradient along y but will be slower than the diffusion transverse to the river (along y).

The dimension of the coefficients K_x, K_y and K_z is a product of the dimensions of the velocity and length, and therefore, assuming that these coefficients are determined by the magnitudes of V and the above expressions for estimating the gradients, we will have

$$K_x = a_x\frac{V}{L_x}L_x^2 = a_xVL_x, \quad K_y = a_y\frac{V}{L_y}L_y^2 = a_yVD, \quad K_z = a_z\frac{V}{L_z}L_z^2 = a_zVh, \tag{2}$$

where a_x, a_y and a_z are dimensionless numerical coefficients of the same order of magnitude. In the general case, these coefficients depend on the Reynolds numbers, where ν is the kinematic viscosity coefficient of the water. However, this dependence becomes weak at the high values of these numbers that are characteristic of the conditions in large rivers. We will estimate the numerical values of the parameters a_x, a_y and a_z later when comparing the results of theoretical forecasting with real data from observations.

The relations in system (2) indicate that the tensor of the turbulent diffusion coefficients are considerably anisotropic, as $h \ll D$ (for large rivers, h is of the order of a few or tens of metres, and D is of the order of hundreds of metres or a few kilometres, i.e., $h/D \sim 10^{-2}$). This means that, in reducing Eq. (1) to dimensionless form by substituting $x = L_x\xi$, $y = D\eta$, $z = h\zeta$, $t = T\tau$ and $\upsilon = Vu$ (the quantities ξ, η, ζ, τ and u are all of the order of unity), and in estimating the orders of the quantities in this equation, it is reduced with great accuracy, with $T = h/a_zV$, to the form

$$\frac{\partial C}{\partial \tau} = \frac{\partial^2 C}{\partial \zeta^2}, \quad \text{т.е.} \quad \frac{\partial C}{\partial t} = K_z\frac{\partial^2 C}{\partial z^2}, \tag{3}$$

with boundary conditions of the absence of flow of the pollutant through the bed and surface of the river. This means that diffusion along z occurs much more rapidly than along y or x, and so there is a "fast" time in the problem, of scale $T = T_z \sim h/a_zV_0$. It follows that the process of mixing of the initial polluted liquid volume with the surrounding water will take place in several stages. The initial state – the rapid stage – is "controlled" by Eq. (3). At this stage, the concentrations of the impurity along the z coordinate are evened out if the densities of the pure water and of the polluted water, ρ and ρ_*, are identical, or a different "quasi-equilibrium" concentration distribution along z is formed if $\rho_* \neq \rho$, with the maximum concentration at the river bed if $\rho_* > \rho$ or at the surface of the water if $\rho_* < \rho$. If the difference between ρ and ρ_* is significant, the impurity will either sink (when $\rho_* > \rho$) and move and diffuse very slowly in a thin near-bed layer of water or (when $\rho_* < \rho$) will remain afloat and diffuse in a thin near-surface layer of water. These cases require special quantitative modelling, in particular a separate analysis of cases where the pollutant may be mixed with (dissolved by) the water or may remain unmixed (for example, when this pollutant is oil). However, we will not examine these cases in detail here, but rather confine ourselves to the case where $\rho_* \sim \rho$, when the difference between ρ_* and ρ has no influence on turbulent diffusion. Here, the condition $|\rho_* - \rho|\, gh \ll 0.5\rho V^2$, or $|\rho_* - \rho| / \rho \ll \rho V^2/2gh$, will be adhered to. For example, when $V = 1$ m/s and $h = 5$ m, the condition $|\rho_* - \rho| / \rho \ll 0.01$ is obtained.

At the first stage, which is completed within a time $t_1 = nT_z \sim nh/a_zV$, $n \sim 3$–5 (within this time, the diffusion "front" n times traverses the water layer of height h), diffusion occurs, of course, in the x and y directions, and this diffusion front moves a distance of the order of $l_x \sim \sqrt{K_x t_1}$, $l_y \sim \sqrt{K_y t_1}$, or

$$l_x \sim \sqrt{a_x V L_x nh/a_z V} = \sqrt{(a_x/a_z)nhL_x}, \; l_y \sim \sqrt{a_y V L_y nh/a_z V} = \sqrt{(a_y/a_z)nhL_y},$$

so that the following estimates are obtained

$$l_x/L_x \sim \sqrt{n(a_x/a_z)h/L_x} \ll 1, \; l_y/L_y \sim \sqrt{n(a_y/a_z)h/L_y} \ll 1,$$

i.e., the dimensions of the region along x and y into which the pollutant diffusing along x and y has penetrated are small compared with the characteristic dimensions of the problem in these directions. The same quasi-equilibrium impurity concentration distributions along x and y will be different for different x and y – they will depend on x and y as on parameters and change slowly during diffusion along x and y at the following stages.

At the second stage, the slower stage, quasi-two-dimensional diffusion occurs along y and x, which ends with the concentrations evening out along the y coordinate. At this stage, the process of diffusion is described by a two-dimensional equation (along x and y) obtained from Eq. (1) by discarding the final term on the right-hand side, or by averaging Eq. (1) with respect to z (the result is the same). The characteristic time of completion of this stage is estimated by the expression $t_2 = nT_y \sim nD/a_yV$, $n \sim 3$–5 (the diffusion front crosses the width D of the river n times).

On completion of the second stage, the third stage, the longest, begins. In this stage diffusion of the impurity occures along the x coordinate, along the river, that is no longer subject to the constricting influence of the boundaries of the water flow. Here, the diffusion process is again described by a one-dimensional equation obtained from Eq. (1) by averaging over y and z, i.e., over the cross-section of the flow, which yields, with a good approximation,

$$\frac{\partial \bar{C}}{\partial t} = K_x \frac{\partial^2 \bar{C}}{\partial x^2}, \quad \bar{C} = \frac{1}{F}\int_F C dF, \tag{4}$$

where F is the cross-section area of the flow.

Equation (4) is obtained from Eq. 1 without the term $\upsilon \partial \bar{C}/\partial x$ because υ is the deviation of the longitudinal velocity of the water from its average value V, so that

$$\frac{1}{F}\int_F \upsilon \frac{\partial C}{\partial x} dF = \frac{\partial \bar{C}}{\partial x}\frac{1}{F}\int_F \upsilon dF + \frac{1}{F}\int_F \upsilon \frac{\partial}{\partial x}(C - \bar{C}) dF \approx 0,$$

since

$$\int_F \upsilon dF = 0 \quad \text{and} \quad \int_F \upsilon \frac{\partial}{\partial x}(C - \bar{C}) dF \approx 0$$

Thus, the process consists of three successive, increasingly long stages in which we can estimate the quantities of interest to us from three different models. The same stages will arise in the same sequence (with some quantitative differences at the first and second stages) when a tributary joins a larger river, with sudden changes in the characteristic parameters of the problem, V, D and h, or, conversely, when a river "takes in" tributaries, likewise with sudden changes in V, D and h. It is not possible, in the fairly rough approach adopted here, to take into account smoother changes in D and h (and consequently in V), which always occur in reality, and consider V, D and h to be constant (with respect to s, and therefore with respect to x) on segments of the river between cross-sections where sudden changes in these parameters occur. To describe the evolution of the process downstream, the estimates made above can be used as the points where sudden changes in V, D and h occur, taking into account the fact that, at these points, the incoming volume of polluted water is no longer a "point" volume but, rather, has been formed at preceding stages of the process and has a finite extent along x that may be significant compared with the parameters D and L_x. However, the process at the third stage after the sudden change in the "initial" data can be analysed by the scheme given below for the third stage of the process before any sudden changes have taken place.

Before turning to a detailed quantitative description of the process at the third stage, which determines the effects of pollution of the river water at considerable distances (compared with D and L_x) downstream, we will remark that there is no need to construct detailed models of turbulent diffusion at the first two stages; for these stages, the orders of magnitude of the quasi-equilibrium concentrations can be estimated from simple integral relations following from the balance of the mass of "preserved" pollutant.

If the characteristic dimensions of the volume of polluted liquid at the initial instant of time are denoted by A_{x0}, A_{y0} and h_0, and the pollutant concentrations in the liquid by C_0, then an approximate relation expressing the above-mentioned balance for the first stage of the process can be written in the form

$$C_0 A_{x0} A_{y0} h_0 = C_1 (A_{x0} + l_x)(A_{y0} + l_y) h, \tag{5}$$

where l_x and l_y are the increments in the horizontal dimensions A_{x0} and A_{y0} of the polluted volume, and C_1 is the characteristic magnitude of the pollutant concentration that arise by the end of the first stage. These increments are estimated by the relations already given above

$$l_x \approx \sqrt{K_x t_1} \approx \sqrt{\frac{a_x}{a_z} nhL_x}, \quad l_y \approx \sqrt{K_y t_1} \approx \sqrt{\frac{a_y}{a_z} nhD}, \tag{6}$$

and therefore, to assess C_1, Eqs (5) and (6) yield

$$C_1 \approx C_0 \frac{h_0 A_{x0} A_{y0}}{h\left(A_{x0} + \sqrt{\frac{a_x}{a_z} nhL_x}\right)\left(A_{y0} + \sqrt{\frac{a_y}{a_z} nhD}\right)}. \tag{7}$$

In this formula, for L_x we can adopt the estimate $L_x = L_{x1} \approx L_y = D$, since the diffusion along x under conditions where the diffusion front has yet to traverse the width of the river does not differ significantly from the diffusion along y.

The concentration by the end of the second stage will be estimated as follows. Within a time of the order of t_{20}, determined from the relation

$$D \approx \sqrt{K_y t_{20}}, \quad \text{or} \quad t_{20} \approx \frac{D}{a_y V}, \tag{8}$$

the diffusion front will traverse the river width D. Then, within a time of the order of $(n-1)t_{20}$, the diffusion front will traverse the river width (there and back) a further $n-1$ times. Within the full time $t_2 = nt_{20}$, along the river the diffusion front will travel a distance

$$l \approx \sqrt{K_x n t_{20}}. \tag{9}$$

As a result, in the volume affected by diffusion, with dimensions along x of the order of l, and along y and z of the order of D and h, a characteristic concentration C_2 is established that corresponds to the end of the second stage. Thus, the balance of the mass of the pollutant for this instant of time will be written in the form

$$C_0 A_{x0} A_{y0} h_0 = C_2 Dhl. \tag{10}$$

In formula (9), the parameter K_x will differ (because of L_x) from that for the first stage, as the diffusion along x under conditions where diffusion transverse to the river has already covered its entire width will differ from the diffusion at the first stage. Therefore, it can be assumed that, in formula (9), the following relation is acceptable for K_x

$$K_x = K_{x2} = a_x V L_{x2} \approx a_x V k D, \quad k \sim 2-6. \tag{11}$$

In this way, relations (9) and (10) are reduced to the form

$$l \approx \sqrt{\frac{a_x}{a_y} kn}\, D, \tag{12}$$

$$C_2 \approx C_0 \frac{h_0}{h} \frac{A_{x0} A_{y0}}{D^2 \sqrt{\frac{a_x}{a_y} kn}}, \quad k \sim 2-6, \quad n \sim 3-5. \tag{13}$$

Formulae (12) and (13) enable us to estimate the longitudinal dimension of the polluted volume and the characteristic magnitude of the pollutant concentration within it by the end of the second stage.

When $\rho_* = \rho$, it can be assumed that $(\max C)_1 = C_1$ and $(\max C)_2 = C_2$, whereas, when $\rho_* \neq \rho$, $(\max C)_1 = k_1 C_1$ and $(\max C)_2 = k_2 C_2$, $k_1 > 1$ and $k_2 > 1$. The quantities k_1 and k_2 are defined by the form of the quasi-equilibrium concentration profiles at the end of the first and second stages and depend on the ratio ρ_*/ρ.

For a quantitative description of the third stage of the process, we must construct a solution of Eq. (4) with a certain initial concentration distribution $C(x,t_2) = C_0(x)$ arising by the end of the second stage and concentrated in an interval of the x axis that has a length defined by relation (12). For the third stage we can consider the pollutant concentration in be quasi-equilibrium along y and z in all cross-sections of the current, i.e., homogeneous over the cross-section, and therefore, in Eq. (4) and below, instead of $\bar{C}$, we can write C.

As the characteristic distances L along the river, for which the evolution of the process in time and along the x coordinate must be calculated, are considerably greater than this initial length, i.e., $L \gg \sqrt{(a_x/a_y)knD}$, the "initial" mass of pollutant can be considered to be concentrated at a "point", i.e., Eq. (4) can be solved with the total mass of pollutant $m = \rho_* C_0 A_{x0} A_{y0} h_0$ specified to be constant throughout the process. However, when formulated in this way, the problem becomes self-similar, and its solution describes well the solution of the problem for a non-point initial distribution with $L \gg l$, which is just what we need for quantitative estimates under the adopted rough representation of the evolution of turbulent diffusion.

The solution of the self-similar problem has the form

$$C = \frac{A}{2\sqrt{K_{x2} t_3}} f\left(\frac{x}{2\sqrt{K_{x2} t_3}}\right), \quad t_3 = t - t_2, \tag{14}$$

where A is a constant and the function $f(x)$ is determined by solving the ordinary differential equation (obtained from Eq. 4 with representation (14))

$$f'' + 2(\xi f' + f) = 0 \tag{15}$$

provided $f'(0) = 0$, and has the form

$$f(\xi) = f_0 \exp(-\xi^2), \quad f_0 = \text{const}. \tag{16}$$

As the solution must satisfy the condition of retention of a pollutant mass equal to the initial mass,

$$m = \rho_* Dh \int_{-\infty}^{+\infty} C(x, t_3)dx = \text{const} = \rho_* C_0 A_{x0} A_{y0} h_0, \tag{17}$$

we finally obtain

$$C = C_0 \frac{h_0 A_{x0} A_{y0}}{2\sqrt{\pi} Dh \sqrt{4K_{x2} t_3}} \exp\left(-\frac{x^2}{4K_{x2} t_3}\right). \tag{18}$$

In this formula, the time t_3 is measured from the end of the second stage, $t_3 = t - t_2$, and the formula can be used when $t_3 \gg t_2$, i.e., t can be written instead of t_3; as noted, this is an asymptotic solution worked out from the initial data arising at the end of the second stage when the polluted volume is quite far from the point where chemical spill into the river occured. Substituting $Vt_3 = L$ into Eq. 18 we obtain

$$C = C_0 \frac{h_0}{h} \frac{A_{x0} A_{y0}}{4\sqrt{\pi} D \sqrt{a_x k DL}} \exp\left(-\frac{x^2}{4a_x k DL}\right). \tag{19}$$

where L is the distance travelled downstream by the "centre" of the polluted volume (where $x = 0$).

From Eq. (19) we obtain a very simple relation for the law of attenuation of the maximum pollutant concentration with distance

$$(\max C)_L = (\max C)_{L_0} \sqrt{\frac{L_0}{L}}, \tag{20}$$

by which it is possible, having data from measurements at a certain L_0, to forecast the max$C$ values for other L values (further downstream). It is remarkable that the decrease in concentration (2) does not depend on any of the parameters of the problem!

From Eq. (19) it is also established that the effective length λ (along the river) of the volume of polluted water changes during its motion downstream when $L \gg l = \lambda_0$ as follows:

$$\lambda = 2\xi_* \sqrt{4a_x k DL}, \quad k \sim 2 - 6, \tag{21}$$

where ξ_* is determined by the adopted range of variation in x ($-x_* \le x \le x_*$) in Eq. (19), taken as $\lambda = 2x_*$, say $\xi_* = \sqrt{2}$ when $\exp(-\xi_*^2) \approx 0.1$, i.e., by the relation

$$\lambda(L) = \lambda(L_0)\sqrt{L/L_0}. \tag{22}$$

Relations (21) and (22), like relation (20), can also be compared with data from observations to check the adquacy of the model of the quantitative process constructed here, and also to forecast the evolution of the parameters of the polluted volume as it drifts downstream. By comparing formulae (19) and (22) with data from observations, we can also determine the constant a_x in formula (11) for the effective longitudinal coefficient of turbulent diffusion.

We will now compare the results of estimates by the proposed model with data from observations for the incident on the Songhua River at the end of 2005 and determine the corresponding constants occuring in the model. First of all, we will substitute into relations (2) the characteristic values of the parameters of the problem for the rivers Amur and Songhua in order to estimate the orders of magnitude of K_x, K_y and K_z. For the Amur, data can be found in Ref. 6, in particular for the Khabarovsk location, $V \sim 1$ m/s, $D \sim 1$–2 km, $h \sim 10$ m. Therefore, in estimates of the orders of magnitude of these parameters, with $a_x \sim a_y \sim a_z \sim 0.1$–1.0, we obtain

$$K_x = a_x V L_x = a_x V k D \sim (0.1 - 1.0)100(2 - 6)(1 - 2)10^5 \text{ cm}^2/\text{s} \sim 10^7 - 10^8 \text{ cm}^2/\text{s},$$

$$K_y = a_y V D \sim (0.1 - 1.0)100(1 - 2)10^5 \sim 10^6 - 10^7 \text{ cm}^2/\text{s},$$

$$K_z = a_z V h \sim (0.1 - 1.0)100 \cdot 10^3 \sim 10^4 - 10^5 \text{ cm}^2/\text{s}.$$

The values obtained cover exactly the range of A_L values for the turbulent transfer of a pulse that is given in the table on page 386 of Monin's book,[5] where, apropos of these A_L values, it is stated: "The coefficient of horizontal turbulent exchange is unknown in advance, and it can range from 10^5 to 10^8 cm/s". But nothing is said of how this range was established. Apart from A_L, the table contains values of the length parameter L_υ, which characterizes a certain internal dimension of the turbulent process. These values range from 8 to 80 km, corresponding to the range of A_L given there, i.e., for an increase in A_L by three orders of magnitude there is an increase in L_υ by one order of magnitude – $A_L \sim L_\upsilon^3$.

Unfortunately, we have been unable to find the values of V, D, h and the longitudinal dimensions of the river channel between characteristic points, in particular between Jilin and the point at which the Songhua flows into the Amur. However, we do have detailed hydrological data for the Amur, kindly gathered, at my request, by Academician M. G. Khublaryan. These data were obtained at a number of locations in 1955–1958 and contain the values of the breadth, depth and cross-section area of the flow and the average and maximum flow rates of water as a function of the time of year. It is clear from these data that the flow rate is at its maximum in the summer (at the beginning of autumn) and falls almost by an order of magnitude at the end of the year and the beginning of the next. The accident in Jilin occurred on 13 November 2005, and the polluted volume evolved against a background of minimal flow rates in the Songhua, in which the variability of the hydrological conditions, it is natural to assume, are the same as in the Amur. It is known that the flow rate of water in the Songhua

is a third of that in the Amur at the point where they meet, and therefore, bearing in mind the data for Songhua, we chose one of the locations of instrumental recording of the hydrological parameters of the Amur with a flow rate of the same order of magnitude. This site was the village of Pompeyevka, and for it the average values in 1955–1958 to the year end were as follows: $h \sim 3$–6 m, $D \sim 450$–500 m, $V \sim 0.2$–0.35 m/s, $Q \sim 200$–1000 m^3/s and $F = Dh \sim 1750$–3000 m^2.

The maximum values of these parameters relate to 1955, when the river was ice-free, while for the other three years it was frozen over, and the average data for these years correspond to lower values of the parameters. Therefore, for subsequent calculations, we will adopt the following average values for the Songhua: $h = 4$ m, $D = 500$ m, $F = 2000$ m^2, $V = 0.2$ m/s and $Q = VF = 400$ m^3/s.

Information was posted on the Internet (1 December 2005) that a chemical slick of 80 km length was approaching the town of Harbin, 380 km away from Jilin, at the start of December, and that it would soon pass Harbin. If it was assumed that, for the middle of the chemical slick (where the pollutant was at its highest concentration) to pass through Harbin, it would have to travel 380 km, and for the entire slick to pass through Harbin a further 0.5×80 km, then the time for this to occur, with $V = 0.2$ m/s, would amount to $t = T = 420 \times 10^3$ m/0.2 m/s $\approx 2 \times 10^6$ s ≈ 24 days. The maximum concentration would pass through at a time $t = T_0 = 380 \times 10^3$ m/0.2 m/s ≈ 22 days. Thus, the adopted value of $V = 0.2$ m/s was in good agreement with estimates by Chinese experts of the time when the chemical slick would pass through Harbin (on 4, 6 and 8 December 2005). This validates our choice of the Amur at Pompeyevka as an analogue for the Songhua.

Below, using these data of Chinese experts, we will take $\lambda = 80$ km in formula (21), and also $D = 500$ m, $L = 380$ km and $\xi_* = \sqrt{2}$, i.e. we will assume that $80 = 4\sqrt{2a_x k \times 0.5 \times 380}$, from which we obtain $a_x k = 40/38 \approx 1$, and, for the range $k \sim 2$–6 adopted above, we obtain $a_x \sim 0.17$–0.5. For the average value $k = 4$, we obtain $a_x \sim 0.25$. These values for a_x lie in the range 0.1–1.0 which, as suggested above, contains the values of a_y and a_z as well. To estimate these parameters, we will calculate the values of t_1, t_2 and l, which are defined by the formulae introduced above

$$t_1 = \frac{nh}{a_z V}, \quad t_2 = \frac{nD}{a_y V}, \quad l = \sqrt[4]{\frac{a_x}{a_y}} kD, \quad n \sim 3\text{–}5, \quad k \sim 2\text{–}6. \tag{23}$$

We will assume, as before, that $h = 4$ m, $V = 0.2$ m/s and $D = 500$ m, and also use the estimate obtained above $a_x k \approx 1$, and for $a_y \sim a_z \sim 0.1$–1.0 we will obtain $t_1 \sim (3$–$5)\ 4$ m/$(0.1$–$1.0) \times 0.2$ m/s ~ 60–1000 s ~ 1–17 min. With $n = 4$ and $a_z \sim a_x \approx 0.25$, we obtain $t_1 \approx 320$ s ~ 5 min. For t_2, similarly we obtain $t_2 \sim 140$–2400 min ~ 2 h 20 min–40 h. With $n = 4$ and $a_y = 0.25$, we obtain $t_2 \sim 700$ min ~ 17 h. From these estimates it follows that, obviously, it is preferable to adopt $a_y \sim a_z \sim 0.1$ and $n = 5$, and here the durations of the first and second periods amount to $t_1 \approx 17$ min and $t_2 \approx 40$ h.

For l we obtain

$$l \approx \sqrt[4]{\frac{n}{a_y}} D \sim \sqrt[4]{\frac{3-5}{0.1-1.0}} 500 \text{ м} \sim \sqrt{3} \cdot 500 - \sqrt{50} \cdot 500 \text{ м} \sim 800 - 2000 \text{ м}.$$

With $n = 4$ and $a_y = 0.25$, we obtain $l \sim \sqrt{4/0.25} \times 500$ m $= 2000$ m. To assess l, it is again preferable to take $a_y \sim 0.1$ and $n = 5$, which yields $l \approx 3.5$ km.

One comment must be made here. In deriving the formulae for t_1 and t_2, the time of single traversal by the diffusion front of dimensions h and D was simply increased. Another approach is possible, by which the times for this front to travel the lengths nh and nD are taken as t_1 and t_2. This will lead to the formulae

$$\bar{t}_1 = nt_1, \quad \bar{t}_2 = nt_2, \quad n \sim 3 - 5, \tag{24}$$

and for $\bar{l}$ we will obtain

$$\bar{l} = \sqrt{n} l, \quad n \sim 3 - 5. \tag{25}$$

Estimates by means of these formulae for the version with $a_y \sim a_z \sim 0.1$ and $n = 5$ yield $\bar{t}_1 = 5 \times 17$ min $= 85$ min $= 1$ h 25 min, $\bar{t}_2 = 5 \times 40$ h $= 200$ h ≈ 8 days and $\bar{l} = \sqrt{5} \times 3.5$ km ≈ 8 km. With $a_y \sim a_z \sim a_x \approx 0.25$ and $n = 4$, we obtain $\bar{t} \approx 4 \times 5$ min $= 20$ min, $\bar{t}_2 \approx 4 \times 17$ h ≈ 70 h and $\bar{l} \approx \sqrt{4} \times 2$ km $= 4$ km.

These estimates are possibly closer to the truth. Nevertheless, for more confident conclusions, more complete and detailed data from observations and full-scale measurements are needed.

We will now estimate the characteristics of the process of the fall in the concentrations of pollutant in the slick as it moves downstream. The initial volume of the pollutant was 100 m^3 of liquid. In accordance with this, we will adopt the values $A_{x0} = A_{y0} = 7$ m and $h_0 = 2$ m. Then, by means of formula (7) we will, with $a_x = 0.25$, $h = 4$ m, $L_{x1} = D = 500$ m, $n = 4$ and $a_y = a_z = 0.25$,

$$\frac{C_1}{C_0} \approx \frac{100}{4 \cdot 500 \cdot 500\sqrt{4 \cdot 4}} = \frac{100}{4 \cdot 10^6} = 2.5 \cdot 10^{-3}.$$

For C_2, from formula (13) we obtain

$$\frac{C_2}{C_0} \approx \frac{98}{4 \cdot 97 \cdot 97} \approx 2.5 \cdot 10^{-5}.$$

Formula (19) can be presented in the form

$$C = \kappa C_2 \sqrt{\frac{l}{L}} \exp\left(-\frac{x^2}{4a_x k D L}\right) \equiv \max C \cdot \exp(-\xi^2),$$
$$\kappa = \frac{1}{4\sqrt{\pi}} (a_x k a_y / n)^{-1/4}. \tag{26}$$

For $a_x k = 1$, $a_y = 0.1$ and $n = 4$, we obtain

$$\max C \approx 0.35 \sqrt{\frac{l}{L}} C_2, \tag{27}$$

and with $L \approx l$ we obtain $\max C \approx 0.35 C_2$ instead of $\max C\,(L = l) = C_2$, which is entirely acceptable considering the inapplicability of asymptotic formula (26) when $L = l$ and the approximate nature of the formula for C_2. For the Harbin site, where $L \approx 400$ km, from expression (27) we obtain, with the above estimates of $l \sim 3$–9 km, $\max C(L = 400) \sim C_2 \sqrt{4/400} = 0.1 C_2$.

For $L = 10^3$ km we obtain $\max C\,(L = 10^3) \approx 0.002 C_2$. Thus, according to the proposed model, the main dilution of the impurity, of the order of 10^{-5}, occurs at the end of the second stage of the process, but further "dimention" of the pollutant concentrations as the slick moves downstream occurs slowly (according to formula (20)), like the very process of "spreading" of the slick (according to formula (22)).

After the pollutant has entered the Amur, at the first and second stages of the start of its motion along the river, its main dilution will occur, and the polluted volume will increase by a factor of $(D_A/D_S)^2 h_A/h_S$ (the subscripts A and S denote whether the parameters belong to the Amur or the Songhua), i.e., roughly by a factor of $3^2 \times 2 = 18$, so that, for $\max C_{2A}$, we obtain the estimate $\max C_{2A} \approx \max C_{2S}\,(L = 10^3)/18 \approx 0.02 C_{2S}/18 \approx 10^{-3} C_{2C} \approx 2.5 \times 10^{-8} C_0$.

It follows from these estimates that the incident on the Songhua/Amur at the end of 2005 should not have serious consequences for the Amur, as was indeed the case. Nevertheless, an ecological catastrophe may occur even with low downstream concentrations in large rivers through the mass asphyxiation of aquatic life in their upper reaches and tributaries.

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